PRIMARY AND REMOTE FRACTURE AROUND UNDERGROUND CAVITIES

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SUMMARY

The development of fractures around cavities in rock is examined using physical and numerical models. In uniaxial compression, fracture proceeds from primary through remote to compressional (slabbing) fracture. There is a wide stress interval where the only event is the stable propagation of the primary fracture. When the crack length becomes equal to about the size of the diameter of the cavity, the critical stress condition shifts to either side of the primary fracture, leading to the nucleation of remote fractures. Almost simultaneously, failure at the compressive stress concentration follows. With increasing stress, fracture in the remote region is seen to extend away from and towards the compressive region, eventually merging with the slabbing fractures. Increasing the confining pressure shortens the stress interval during which primary fracture is the only process and shifts failure to the remote and the compressional fracture zone.

INTRODUCTION

Whether they occur around mine openings, underground civil engineering installations, nuclear waste repositories or other underground excavations, the formation of fractures around underground workings is of considerable interest. Fracture may manifest itself in various degrees of intensity, ranging from harmless spalling at the excavation perimeter to the explosive violence of rock bursts. Although rock failure of all types has been experienced through the centuries, it is only recently that the attention of the rock mechanics research community has been focused on the problem. Still, very little information can be found in the literature about the geometry, the mechanics and the stress condition associated with large-scale rock failure. In simple laboratory loading situations, on the other hand, the evolution of fractures around physical models of cavities is relatively well known. Under increasing uniaxial compression (axial stress), and at relatively low confining (lateral) stress, a block of rock containing a circular model tunnel will in general show a slow, stable evolution of fracture:

1. primary tensile fracture (P)
2. secondary or remote fracture (R), and
3. side-wall slabbing or compressional failure (S).

The primary tension fractures have long been recognized, since they form in response to a predictable tensile stress. Similarly, failure in the compressional zone of the rock wall perimeter is expected from the presence of the high compressive stress concentration. The origin of the secondary, or remote, fractures is less obvious. As the name implies, they form remote from the opening, and therefore may not become visible in underground workings. Their existence has, however, been demonstrated through physical model tests by Hock, Lajtai, Gay and Ingraffea. Using a photoelastic coating technique, Hock traced the origin of remote fracturing...
to the tensile stress concentration that forms remote from the opening as the first developing primary tensile fracture increases in length. The tensile stress at this remote point may become large enough to nucleate and then propagate a tensile fracture in a stable, load-controlled manner. Ingraffea\textsuperscript{2} analysed Hock's results, using a finite element code (employing linear elastic fracture mechanics, LEFM) and showed that tension is created in the remote region and causes fracturing. Using plaster models, Lajtai and Lajtai\textsuperscript{6} demonstrated that the remote fractures combine ultimately with the failure process in the compression zone (slabbing-crushing and shear fracture in plaster) causing the collapse of the cavity.

The motivation for this physical/numerical modelling of remote fracture nucleation has come from observing the fractures around collapsed underground workings in the Cominco potash mine in Vansoy, Saskatchewan. Many of the potash mines of Saskatchewan operate under conditions of stress, high enough to cause some degree of slabbing and extensive fracture in the yield pillars. Recent re-excavation of older abandoned parts of the mine exposed several sections of collapsed workings and fractures in and above the pillars between them. Many of the fractures had formed remote from the openings and bore some resemblance to the remote fractures of the physical model tests.

In this investigation, another small step is made towards a better understanding of the way primary and remote fractures form in the mining environment. This is accomplished through the presentation of the results of a combined physical and numerical modelling exercise that was aimed at defining the fracture process around a cylindrical cavity in detail, starting from the point of primary fracture initiation and finishing at the appearance of the first remote fracture. The physical modelling used Lac du Bonnet granite for which a substantial database is already available, backed up by a more limited exercise using both potash and anorthosite. Loading was in uniaxial compression only. The numerical modelling, on the other hand involved a programme of finite element analysis that covered a much wider range of stress conditions.

**PHYSICAL MODELLING**

Physical testing for the influence of holes on the strength of rocks and other engineering materials has a long tradition. The problem and its solution are relevant in two widely different areas of concern. In (a) brittle fracture mechanics, material flaws in the form of elongated voids (Griffith cracks) of microscopic scale are considered as stress raisers at which microfractures may nucleate. In (b) mining and tunnelling operations, the underground openings (megascopic scale) become the 'flaws' in the rock mass that in turn may have its own microscopic (voids, vugs, pores, etc.) and macroscopic flaws (joints, bedding, foliation, solution cavities, lava tunnels, etc.). Neither the microscopic nor the megascopic size is convenient in physical models, and therefore the mechanics of fracture at both scales is evaluated through physical models that are largely of macroscopic size. The outcome of the modelling exercise may then be directed towards either concern, provided that the consequences of the drastic change in size and the closely related stress gradient effect are carefully evaluated.

It is worth while to note that the results of physical modelling are often interpreted differently in the two areas of concern. In brittle fracture mechanics, most investigators ignore failure in the compressional zone of the void or crack entirely, and only a few (e.g. Ingraffea\textsuperscript{4}) recognize the contribution of remote fractures. Conversely, in the mining-orientated investigations, it is failure at the compressive stress concentrations that is of major concern. There is, however, a good reason for this. In mining and other tunnelling operations, primary tensile fractures are probably rare, since the confining pressure is usually high enough to suppress the tensile stress concentration at
the cavity wall. At the mining scale, secondary or remote fractures are not well known either as they rarely intersect the cavity perimeter.

Since the evolution of fracture around a circular cavity loaded in uniaxial compression is at least qualitatively well established, the physical experiment was designed to provide quantitative data on the nucleation of the primary, the remote and the compressional fracture processes. For this purpose, a $200 \times 200 \times 60$ mm block of granite was cut and ground to accurate size. A 36 mm hole was then drilled in the centre of the large face parallel to the least dimension. The instrumentation consisted of 19 strain gauges with a 5 mm long measuring grid that were bonded to the inside of the hole and to the surface of the block in positions where the primary, the remote, and the compressional fractures were expected to appear first (Figure 1). The actual locations were picked based on the experience gained in two earlier tests, using this granite in one and Bleebe anorthosite in the other. In order to minimize friction at the rock-loading platen interface during the uniaxial compression test, two 30 mm-thick granite loading platens were machined to match the size of the loading surface of the rock block. These were then inserted between the rock block and the loading platens of the testing machine. With the strain gauges connected to a data-acquisition system, the block was loaded in a 600,000 lb capacity testing machine to reach 157 MPa (load divided by the full cross-sectional area), producing an average 'pillar stress' (load divided by the net cross-section through the centre of the hole) of 194 MPa. The mean uniaxial compressive strength of intact Lac du Bonnet granite is 226 MPa (Table 1). At the 157 MPa load, all three elements of the fracture process were clearly displayed. Loading was stopped before collapse could occur in order to save the specimen for visual examination (Figure 2).

The strain gauges were strategically positioned to signal the nucleation of the three fracture processes. In fact, the gauges positioned to signal remote fracture (e.g. gauge 7) would typically respond to primary fracture as well (Figure 3). Fracture at a particular gauge location is reflected through the deflection of the load–strain curve. A fracture crossing under the gauge shows rapidly

![Figure 1. The granite test block with the strain gauge instrumentation. The gauges shown inside the hole were bonded to the inside wall. Loading was in uniaxial compression.](image-url)
Table 1. Mechanical properties of Lac du Bonnet Granite*

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Number of tests</th>
<th>Value</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tensile strength</td>
<td>66</td>
<td>14 MPa</td>
<td>1.2 MPa</td>
</tr>
<tr>
<td>Uniaxial compressive</td>
<td>28</td>
<td>226 MPa</td>
<td>15 MPa</td>
</tr>
<tr>
<td>compressive strength</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Modulus of elasticity</td>
<td>15</td>
<td>71.3 GPa</td>
<td>4.7 GPa</td>
</tr>
<tr>
<td>Fracture toughness†</td>
<td>15</td>
<td>2.45 MPa√m</td>
<td>0.2 MPa√m</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>15</td>
<td>0.25</td>
<td>0.04</td>
</tr>
</tbody>
</table>

Triaxial strength parameters (426 tests) $m = 31$ ($s = 1$), $σ_3 = 245$ MPa‡

* Lac du Bonnet quarry.
† From the double tension test.
‡ Obtained from curve fitting to data from 426 tests.

Figure 2. The test block after a load of 157 MPa. The fractures were ‘inked-in’ for emphasis

increasing extensional strain (e.g. gauge 4 in Figure 4). Conversely, a fracture approaching and then bypassing the gauge leads to stress relief (e.g. gauges 1 and 3 in Figure 4). Gauges positioned in the compressional zone do not seem to react to fracture occurring in the primary and remote positions. They signal only the ‘yielding’ at the compressive stress concentration and then the relaxation of the compressive strain as the crushed rock sliver separated from the rock wall.

The 19 strain gauges and the load-strain curves derived from them yielded 12, 13 and 5 measurements for primary, secondary and compressional fracture nucleation, respectively (Table
Figure 3. A typical axial load versus local strain curve. This gauge was located in the remote fracture zone. The acceleration of extensional strain marks the appearance of the remote fracture. The deflection at the point of primary fracture suggests stress and strain relief in the shadow of the passing fracture.

Figure 4. Three load versus local strain curves, indicating the relationship between strain and the fracture path. Rapid increase in strain marks the appearance of a ‘through-fracture’ at gauge 4. The fracture zone stabilizes, however, with little change occurring after about 25 MPa. Bypassing the gauge, at gauges 3 and 1, has the opposite effect; stress relief through fracture limits the rate of strain produced at gauge 3 and decreases it at gauge 1.
II) with corresponding mean values of 16, 119 and 126 MPa. The comparable stress figures from the earlier conducted Babulic experiment on granite were somewhat lower: 15, 78 and 92 MPa, suggesting that these values are subject to the usual statistical variation and can change from block to block and from investigator to investigator.

The fracture process followed the expected pattern. The primary fractures formed at 16 MPa, that is 2 MPa higher than the Brazilian tensile strength of this granite (Table I). Theoretically, using the Kirsch solution and the maximum stress criterion, this should have occurred at 14 MPa. The higher load necessary to nucleate the primary fracture suggests that the stress-gradient effect is noticeable at this scale. The typical primary crack propagated to about 43 mm at 135 MPa and extended another 4 mm by the end of the test, at 157 MPa. The secondary, or remote, fractures made their appearance at 119 MPa extending with increasing load both away from and towards the cavity. One important characteristic of remote fracture is that it is not a single crack, but a fracture zone that may consist of a whole series of parallel fractures (Figures 2 and 5). In fact, they are very similar to the parallel set of axial fractures usually observed in specimens that come from

<table>
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<tr>
<th>Fracture type</th>
<th>Number of observations</th>
<th>Stress (MPa)</th>
<th>Standard deviation (MPa)</th>
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</thead>
<tbody>
<tr>
<td>Primary</td>
<td>12</td>
<td>16</td>
<td>2.7</td>
</tr>
<tr>
<td>Remote</td>
<td>13</td>
<td>119</td>
<td>10</td>
</tr>
<tr>
<td>Slabbing</td>
<td>5</td>
<td>126</td>
<td>12</td>
</tr>
</tbody>
</table>

Figure 5. Remote fracture zone in the anorthosite block. Uniaxial loading from left to right. At high stress remote fractures multiply rapidly to form a fracture zone. Magnification is 10×.
standard uniaxial or triaxial compression tests. Failure in the compression zone appeared in the form of spalling starting at 126 MPa. Since a stress concentration of three should exist at this point, one would expect failure to start at one-third of the uniaxial compressive strength, between 70 and 80 MPa. The relatively late arrival may suggest the influence of the very steep stress gradient at this point.

Besides producing data on fracture nucleation, the experiment yielded two unexpected results that are worthy of comment. During the later stages of fracture evolution, between 140 and 157 MPa, the remote fractures could be seen merging with the fractures of the compression zone (Figure 6). The second observation concerns the appearance of some strange horizontal fractures between the cavity and the subvertical secondary fractures (Figures 7 and 8). These were present in the anorthosite and the earlier granite block as well, so that their formation cannot be dismissed as being accidental. In the experiments of Gay\textsuperscript{3} a whole class of 'domical' fractures were defined, which has not been observed here. On the other hand, one of the inside remote fractures of the granite block tested here (Figure 2), did display a distinct inward curvature.

**NUMERICAL MODELLING**

The numerical modelling exercise had two goals: first to interpret the observed fractures in the physical experiment in terms of the changes in the stress distribution during rising axial load, and secondly to extend the results on remote fracture nucleation to the more general case where a confining pressure is also involved.

![Figure 6. A micrograph (10 x) enlarging the compressional failure zone in the granite test block. Here slubbing merges with the remote fractures. The uniaxial load is from left to right]
Figure 7. The appearance of an unexplainable 'horizontal fracture' in the anorthosite block. Magnification is 5 ×

Figure 8. The 'horizontal fracture' in the granite block at 3 × magnification. It runs between the vertical remote (right) and primary (left) fractures.
The technique

For the numerical analysis, a commercial finite-element package, the PC-Educational version of ANSYS by Swanson Analysis Systems Inc., was used. A two-dimensional, plane strain, four-noded quadrilateral element with linear elastic properties was chosen to model the rock behaviour. The radius of the cavity was 1/15 of the length of the whole model. The mesh consisted of an annular region of 1-2 radius wide, where the mesh size was uniformly 6 per cent of the tunnel radius. This region was then followed by an annulus having a 10 per cent size, and beyond this the element size increased, as shown in Figure 9. During the analysis phase of ANSYS, the displacements and the stress components were computed at each nodal point. The nodal stresses were averaged for each element, and this is the 'mesh stress' used in defining the conditions of fracture in both the primary and the remote fracture locations.

The fracture criteria

The traditional method of stability analysis in rock mechanics follows the 'safety-factor approach', which represents the general equilibrium of a structure. The state of stress is evaluated and the relevant stress parameter is compared with the available resistance. In the method outlined by Hoek and Brown, for example, the maximum principal stress ($\sigma_1$), is compared with the triaxial strength ($S$), of the rock mass:

$$\text{Safety factor} = \frac{S}{\sigma_1}$$

where $S$ is a function of the minimum principal stress ($\sigma_3$) and the rock mass properties. It is evaluated through a combination of tensile (usually the Brazilian) and triaxial compression tests. In this paper, a slightly different terminology is implied, where the 'safety-factor' is a measure of fracture stability much like $K_{IC}$ in fracture mechanics.

![Figure 9. The mesh pattern around the hole as used in the finite element analysis](image)
The major problem with the safety-factor approach and in particular with its finite element implementation is the 'scale-dependence'. Both the stress and the fracture strength functions are formulated in terms of parameters that occur 'at a point'. At points where high stress concentrations occur, this formulation leads to unrealistic results. The alternative approach, LEFM as outlined for example by Ingraffea,\(^7\) appears to avoid this problem. In its simplest form, fracture is expected to propagate as long as \(K_i > K_{IC}\), where \(K_i\) is the stress intensity factor for Mode I and \(K_{IC}\) is the fracture toughness. Concerning rock fracture from a circular cavity, the fracture mechanics formulation appears to work because the propagating primary fracture creates a tension zone in the remote region. There is, however, overwhelming evidence that 'tensile' fractures propagate in an 'all-compressive' stress field as well. In fact, the propagation of fractures parallel to the maximum principal stress (compression positive) trajectory, is the dominant mode of fracture in uniaxial and triaxial compression (low confining pressure) of rocks that do not contain a machined void or crack. LEFM needs a tensile stress to propagate cracks and for this reason a 'starting-crack' with its localized tension must be assumed. Since at the microscopic scale, the only evidence for fracture in Lac du Bonnet granite is the presence of cracks running parallel to the compression direction, such a 'starting crack' would have to be of submicroscopic size. In fact, at the microscopic and macroscopic scales and under compressive loads, fracture in rocks is driven by a compressive stress acting parallel to the crack length. The effect of the normal stress parallel to the propagation direction is of course neglected in the fracture mechanics formulation. The consequence of this may not be apparent where the compressive stress is small, as for primary fracture initiation where the radial stress is zero, but for remote fracture where the axial compressive stress is large, the LEFM approach underestimates the condition for fracture. Remote fracture has been shown to nucleate at a tensile stress that is substantially smaller than the tensile strength.\(^3\)

LEFM can only work for remote fracturing by assuming a starting crack of 'substantial size' at the point where the tensile stress is the greatest, as was done by Ingraffea.\(^4\) In our particular case the critical crack length for a tensile strength of 14 MPa (Brazilian test) and fracture toughness \((K_{IC})\) of 2-45 MPa/m (double torsion test), assuming a 'penny-shaped' crack, computes to about 48 mm, an unrealistically large size, since specimens of this dimension were used to determine the tensile strength.

The fracture strength approach may appear crude at first sight, but it has a clear advantage over fracture mechanics in using an empirical strength criterion that incorporates the effect of all stress components, whereas fracture mechanics neglects two normal stress components of the stress tensor. The 'scale effect' can be included by introducing a 'stress-averaging' method as outlined by Lajtai\(^10\) and Nesetova and Lajtai.\(^11\) This can be implemented by 'fine-tuning' the mesh size, until the predictions of the numerical model match the physical experiment. In fact, calibrating against a physical experiment is at present the only way rock fracture in compression can be approach. The strength method allows this since it does not have the requirement of the 'starting-crack'.

For the fracture strength function, the Hook and Brown strength criterion is used. This criterion is actually for failure: fracture propagation starts at a lower stress. However, examination of microscopic thin sections obtained from specimens stressed just below and to failure suggest that most fractures form close to peak strength, thus the error introduced, in view of all other uncertainties, is negligibly small.

Although the 'safety-factor approach' has been used successfully for a long time, most recent studies use fracture mechanics.\(^4\),\(^5\),\(^12\),\(^13\) There are however others who follow some variation of the safety-factor approach (e.g. Kowamoto and Saibo\(^14\)). After the relative advantages and dis-
advantages of proceeding in either way were considered by the authors, the safety-factor approach was chosen.

Around voids in rock, both tensile and compressive stresses are present. Nevertheless, fracture through either type of stress conditions can be represented through a general fracture criterion expressed in terms of at fracture, the safety factor is then the ratio of to the given confining pressure, , the computed maximum principal stress, . for Lac du Bonnet granite can be defined through data from Brazilian tension, uniaxial compression and triaxial compression tests, which were done earlier. Because the Hoek and Brown parabola gives a poor fit to our test results in the tensile region, a second-order polynomial function was fitted to the data covering the tensile and the low confining pressure region, to 50 MPa confining pressure (Figure 10):

\[ S_1 = 240 + 13.6 \sigma_3 - 0.09 \sigma_3^2 \text{ for } \sigma_3 < 50 \text{ MPa} \]

At confining pressures higher than 50 MPa, it is the polynomial function that has the wrong shape. Therefore, the Hoek and Brown parabola was substituted to describe the strength-confining pressure relationship over 50 MPa:

\[ S_1 = \sigma_3 + \sqrt{[7595 + \sigma_3(245)]^2} \text{ for } \sigma_3 > 50 \text{ MPa} \]

Here the value of 7595 is the product of the modulus , which is 31, and , which is 245; both parameters determined through the curve-fitting exercise. The strength of Lac du Bonnet granite

![Graph](image)

Figure 10. The strength function used in the analysis was based on Brazilian and triaxial tests. A polynomial function modelled failure at low (less than 50 MPa) confining pressure, and the Hoek and Brown parabola modelled failure at higher confining pressure.
can be described through these two formulations for a wide range of stress states around an underground opening with one exception. In front of the propagating primary fracture, biaxial tension exists, that is both in-the-plane principal stresses, \( \sigma_1 \) and \( \sigma_3 \), are negative. Because of the uncertainty associated with fracture under biaxial tension, the maximum stress theory was used as the fracture criterion for the propagation of the primary fracture:

\[
S_1 = \frac{\sigma_1}{\sigma_3}
\]

where \( \sigma_3 \) is the largest tensile stress and \( \sigma_1 \) is the tensile strength. For the sign convention used here both parameters are negative.

The analysis

To find the safety factor at any point in the rock around the cavity except at the primary crack tip, the procedure followed was to first generate the principal stresses, \( \sigma_1 \) and \( \sigma_3 \), at that point, compute \( S_1 \) and then compare \( \sigma_1 \) with \( S_1 \) to establish the safety factor. In this two-dimensional analysis, we use \( \sigma_1 \) and \( \sigma_3 \) as the in-plane principal stresses, i.e., it is assumed that \( \sigma_2 \) acts along the axis of tunnel. At the point where the safety factor was found to be less than unity, the remote crack orientated parallel to the maximum principal stress trajectory was assumed to have been formed. The remote fracture was not propagated.

Since the stress condition required for the start of the remote fracture is created through a change in primary crack geometry, it was necessary to model the primary crack propagation process. As the primary fracture propagates away from the cavity, the critical stress region shifts from the tip of the primary fracture to a region on either side of the primary fracture. For a given confining pressure, the primary fracture was nucleated by finding the axial load that produced an \( SF = 1 \) condition at the expected high tension location on the perimeter. The propagation was simulated by removing the displacement constraint at the end of the first node above the opening and thus extending the crack by the mesh size. The stress analysis was then repeated for the new crack geometry. If the safety factor for the next mesh was less than unity, the displacement constraint at the next node was released as well. Otherwise the axial load was increased until the \( SF = 1 \) condition was indicated for the next mesh. This exercise was repeated, until the low safety factor condition shifted to the remote region at which point the analysis for a particular confining pressure stopped. Figure 11, showing the distribution of the minimum principal stress, is an example for this case. Here a tension of 9 MPa exists both at the tip of the primary fracture and in the potential remote fracture location. The tensile strength being 14 MPa, no fracture forms at the crack tip, but in the remote location the relatively high maximum compressive principal stress combines with the tensile principal stress to create an \( SF = 1 \) condition (Figure 12).

The \( SF = 1 \) condition in the remote regions was interpreted to signal that a crack orientated parallel to the maximum principal stress trajectory formed at this point. During primary fracture propagation, the increasing axial stress would typically create a low-safety-factor situation at the compressive stress concentration as well. Our modelling of remote fracture nucleation ignored the potential influence of this condition.

The results

The data collected on the relationship between axial stress and primary crack length at various cases of constant confining pressure are presented in Figure 13. The inserted trend lines, following the data, suggest the expected stress-hardening: larger and larger stress increments are required to extend the crack as the crack becomes longer. The curves, shown for each confining pressure
Figure 11. The distribution of the minimum principal stress at the point of remote fracture nucleation at 156 MPa axial and 30 MPa lateral load. Tension is considered negative here.

Figure 12. The distribution of the safety factor for the case shown in Figure 11.

separately, are terminated where the remote fractures form, and the numerical analysis was stopped. The primary fractures would have travelled beyond this point, but probably at a decreasing rate. Above about 110 MPa confining pressure, i.e., below the axial/lateral stress ratio of 3, primary fracture is negligible.

For all three fracture types, the axial load at the point of fracture nucleation varies linearly with confining pressure (Figure 14). However, the rate of increase is different for each. The confining pressure seems to be the most effective in suppressing the primary fracture and the least effective in the prevention of compressional failure.
Figure 13. The primary crack length at the point of remote fracture nucleation for a range of confining pressures and as a function of axial load.

Figure 14. The criteria for primary, remote and slabbing fracture. For each, the axial load increases linearly with the lateral load, although at different rates.
The location of the starting remote fracture changes with confining pressure (Figure 15). As the confining pressure increases, the fracture moves closer to the perimeter and at the same time acquires a lower dip. The numbers identify the horizontal ($P_H$) and vertical ($P_V$) loads (about 1/6 th of model is shown).

Figure 16. The distance to the point of remote fracture nucleation.

The location of the starting remote fracture changes with confining pressure (Figure 15). On increasing confining pressure, the point of nucleation moves closer to the opening (Figure 16) and at the same time acquires a shallower dip (Figure 17). The starting remote fracture is practically vertical at no confining pressure, but would rotate to a dip of about 60° at high confining pressure. This is the reason why the remote fractures in the granite test block (Figure 7) are all close to
vertical. As the confining pressure increases, the point of remote fracture nucleation moves in toward the cavity following an apparently straight path along the polar angle of 58° ± 1°, measured from the horizontal.

DISCUSSION

The numerical data on primary, remote and compressional failure presented in the previous Figures refer to conditions that would exist around a 36 mm cylindrical cavity located in intact Lac du Bonnet granite. Clearly, the influence of discontinuities, potential anisotropy and other deviations from homogeneity and purely elastic conditions is beyond the scope of this study. In addition to these effects, however, there is still the size effect to consider if the outcome of this investigation is to be applied to larger caverns, even if they are unlikely to be located in elastic, intact granite. The mesh size at 6 per cent of the tunnel radius seems to have produced data that are in broad agreement with the few measurements from the physical experiment. Some adjustments of the results will have to be made, however, for the larger size of underground workings.

The computed mesh stress of the numerical analysis represents the average stress for the element. Therefore, for a stress field that involves a stress-gradient, the fracture nucleation stress established through finite element analysis will change depending on the selected mesh size. Table III gives the intercept (uniaxial compression) and the slope of the various fracture criteria obtained by selecting a mesh size that is 15, 10 and 6 per cent of the cavity radius. For primary and compressional failure, one may rely on the Kirsch solution to obtain the comparable fracture
### Table III. Fracture criteria: dependence on mesh size

<table>
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<tr>
<th>Mesh size, percentage of radius</th>
<th>Primary Intercept</th>
<th>Primary Slope</th>
<th>Remote Intercept</th>
<th>Remote Slope</th>
<th>Slabbing Intercept</th>
<th>Slabbing Slope</th>
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<tr>
<td>15</td>
<td>19</td>
<td>3.5</td>
<td>70</td>
<td>2.7</td>
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<tr>
<td>10</td>
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<td>77</td>
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<td>3.0</td>
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<td>80</td>
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</tr>
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</table>

* Intercept and slope refer to the parameters of the straight line relationship between axial and lateral load representing the criteria for fracture nucleation.
† Slabbing is not yet indicated and the analysis was stopped when remote fracturing was indicated.
‡ Derived from the Kirsch solution. There is no analytical solution for remote fracture.

criteria for an infinitesimal element as well. The mesh size effect seems to be the greatest for compressional and the smallest for the remote fracture. In fact, the 6 per cent mesh is just too coarse for the compression zone to produce parameters that come close to those obtained from theory. The results are close to the experimental results for a 36 mm hole, however, indicating that a very steep stress gradient exists at the compressive stress concentration (side-wall) for both the rock and the finite element mesh. On the other hand, the same parameters for primary and remote data are quite comparable to those of the theoretical solution, reflecting the shallower stress gradient in those locations. The steep stress gradient at the side-wall allows the rock to be stable at loads above the theoretical failure load.

Others (Santarelli et al., Vardoulakis et al.) attribute the ‘abnormal stability’ at compressive stress concentrations to the pressure sensitivity of the elastic modulus of rock, with the ‘scale or size effect’ related to rock microstructure. For the stress range investigated here, however, the elastic modulus of Lac du Bonnet granite shows no appreciable ‘pressure sensitivity’. The stress gradient around the cavity is controlled by both the cavity size and the rock microstructure, which makes it a simple and convenient tool in assessing the scale or size effect, compared to the more complex model of Vardoulakis and Papanastasiou.

The stress gradient is size dependent; over a fixed distance (d—determined by the rock properties), the gradient becomes shallower with increasing cavity size. Therefore, at the scale of mine workings, the stress gradient may vanish, and the proper fracture criteria then become those based on the Kirsch, or theoretical solution. There is no theoretical solution for remote fracture. One could approach one by shrinking the mesh size. However, the mesh size effect for the primary fracture suggests that the change from the 6 per cent mesh to the infinitesimal element size would make a negligibly small difference, and the fracture criteria for the three fracture types would only be marginally different from those shown in Figure 18. The difference between Figure 14 (36 mm hole) and Figure 18 (large scale) suggests that failure in the compression zone could be more extensive at the large scale of the mine environment than one would expect based on the physical model. A substantial amount of slabbing and crushing at the compressive stress concentrations could occur even before the remote fracture is nucleated.

The fracture criterion for remote fracture was developed on the assumption that failure in the compression zone has no influence on the stress distribution in the remote location. In fact, failure through slabbing in the physical model gradually extended the cavity in the lateral (perpendicular to the axial load) direction. By the end of the experiment, the originally circular void was
Figure 18. Figure 14 presented the fracture criteria for the granite test block. For the larger size of underground openings, the size effect would be small. The fracture criteria presented in this Figure could be used when the stress gradient effect is negligible. Note the substantial change in the course of the criterion for slabbing fracture.

Figure 19. The finite element modelling of remote fracture ignored the potential influence of failure in the compressional zone. The effect of slabbing failure would be the widening of the cavity perpendicular to the axial stress. This Figure shows how changes in the shape of the circular cavity would affect the remote fracture criterion. Lateral expansion, for example, would lower the axial stress required for fracture nucleation. The change, however, is quite small.
somewhat distorted and showed a 20 per cent increase in the horizontal axis of the cavity. This
type of lateral mining through stable spalling could become even more important at higher
confining pressure, where failure in compression would dominate the other two fracture modes.

The effect of such lateral expansion has not been analysed specifically, but the significance of the
resulting lateral expansion on remote fracture nucleation can be deduced from Figure 19, which
compares the trends of remote fracture nucleation for three shapes, including the circular. The
nucleation stress is affected somewhat by the shape change from an axis ratio of unity (the circle) to
a ratio of 3 (the vertical and the horizontal ellipses). The stress at nucleation drops slightly when
the cavity is expanded laterally and increases when the expansion is in the vertical direction.
Consequently, failure occurring at the compressive stress concentration does not reduce the
chance for remote fracture nucleation; it does, however slightly, the opposite. Therefore, the
assumption of the linearity of the nucleation stress versus confining relationship is not strictly
correct beyond the point of compressional failure initiation. As the cavity expands laterally, the
slope of the $e_1-e_3$ relationship for remote fracture nucleation should slowly decrease. For the
expected small change in the lateral dimension, the departure from linearity may not, however, be
that significant.

SUMMARY AND CONCLUSIONS

The primary fractures nucleate and propagate in a manner that was described by many other
investigators. The required stress to effect nucleation increases with confining pressure, in a linear
manner. The primary crack length at the point where remote fractures form decreases with
confining pressure from about 2 times the radius of the cavity in uniaxial compression to a
negligibly small length at 100 MPa confining pressure. Primary fracture propagation occurs in a
stable manner; as the crack becomes longer, larger load increments are needed to advance the
radius of the crack by the same amount.

The axial load required to nucleate remote fractures increases with confining pressure, again in
a linear fashion. Not only the stress, but also the location of the newly formed remote fracture
changes with confining pressure. As the confining pressure increases, the point of nucleation
moves closer to the cavity, starting at about 1-4 times the radius at no confining pressure to 0.2
times the radius at 110 MPa confining pressure. The point of nucleation seems to shift along the
polar angle of 58°± 1°. With the shift in position, the initial dip of the resulting remote fracture
changes from subvertical (85° dip) in uniaxial compression to a shallower dip of 55° at 110 MPa
confining pressure.

In uniaxial compression, and at low confining pressure, the usual sequence of the fracture
evolution is from primary through remote to compressional fracture. Because the confining
pressure has the greatest effect on primary fracture nucleation and least on compressional failure,
this sequence may change as the confining pressure is increased. At very large confining pressures,
only side-wall slacking should be possible. The survival of remote fractures in an environment of
increasing confining pressure, above 120 MPa, has not however been investigated.

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