21

Simulating Fully 3-D Hydraulic Fracturing

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21.1 Introduction

Hydraulic fracturing, the process of initiation and propagation of a crack by pumping fluid at relatively high flow rates and pressures, is one of several techniques for creating cracks in rock. Fractures in the earth’s crust are desired for a variety of reasons, including enhanced oil and gas recovery, re-injection of drilling or other environmentally sensitive wastes, measurement of \textit{in situ} stresses, geothermal energy recovery, and enhanced well water production. These fractures can range in size from a few meters to hundreds of meters, and their cost is often a significant portion of the total development cost. In locations where the \textit{in situ} stress field, including the directions, is known and the wellbore is aligned with one of the far-field principal stresses, the hydraulic fracture geometry can be predicted and controlled with reasonable accuracy. For those wellbores that are not aligned with such a direction (deviated wells), the hydraulic fracture geometry is usually more complex and more difficult to model, especially close to the wellbore where the local stress field is significantly different from the far-field stresses. Field data from hydraulic fracturing operations exist primarily in the form of pressure response curves. It is difficult to define the actual hydraulic fracture geometry from this data alone, however. Therefore, numerical simulations are used to evaluate and predict the location, direction and extent of these hydraulic fractures.

Simulations range from two- to fully three-dimensional depending on the degree of complexity of the wellbore and fracture geometries, the capability of the available simulator, and the required accuracy of the predictions. Numerous 2-D, pseudo-3-D, and planar 3-D hydraulic fracturing simulators exist, and these simulators work very well in many cases where the geometry of the fracture is easily defined and constrained to a single plane. However, there are instances where a fully 3-D simulator is necessary for more accurate modeling. For example, fractures from deviated wellbores are generally non-planar with arbitrary crack front shapes. Most hydrofracturing simulators simply ignore the near-wellbore effects of deviated wells and assume a planar starting crack that has
Simulating fully 3-D hydraulic fracturing

21.2 Background

The process of hydraulic fracturing is not new. Nature has produced many such fractures in the earth's crust (see, for example, Bahat, 1991). The first recorded application of hydraulic fracturing for enhancing oil recovery that the authors are aware of was in 1947 in Kansas (see Howard and Fast, 1970). The 'Hydrafract' concept was formalized first by Clark (1949), although others had recognized previously that 'pressure parting' could occur during acid treatment and water injection, a phenomenon considered to be closely related to the 'Hydrafract' concept (Howard and Fast, 1970). The Hugoton field in western Kansas was the site of the first hydraulic fracturing operation, and by the mid-1960s, hydraulic fracturing had become the dominant method of stimulation in this and many other fields, including geomechanics (Howard and Fast, 1970).

During this early period of hydraulic fracturing, two simple models were proposed to try to predict the shape and size of a hydraulic fracture based on the rock and fluid properties, the pumping parameters, and the in situ stresses (Kistner and Zoltewicz, 1955; Geertsma and de Klerk, 1969; Perkins and Kern, 1961; Nordgren, 1972). The models are known as the KGD and PKN models, and their description can be found in the above references and in many other summaries or texts (e.g., Geertsma, 1989; Mendelson, 1984a,b). Perkins and Kern (1961) and Geertsma and de Klerk (1969) also derived a model for radial hydraulic fracturing. The radial, KGD, and PKN models are essentially two-dimensional plane strain formulations with fluid flow only along the length (or radius) of the fracture. The fracture width and shape are related to the fluid pressure distribution in the fracture; the KGD model has a constant height and constant width through the height, while the PKN model has a constant height and an elliptical vertical cross-section.

The 2-D models are not able to simulate both vertical and lateral propagation. Therefore, pseudo-3-D models were formulated by removing the assumption of constant and uniform height (Settari and Cleary, 1986; Morales, 1989). The height in the pseudo-3-D models is a function of position along the fracture as well as time. The major assumption is that the fracture length is much greater than the height, and an important difference between the pseudo-3-D and the 2-D models is the addition of a vertical fluid flow component. The pseudo-3-D models have been used to model fractures through multiple rock layers with differing stresses and properties. These models are simple, fast, and relatively effective. Wapinski et al. (1994) recently provided brief descriptions and a comparison of predictions for a number of simulations, including 2-D and pseudo-3-D models.

Pseudo-3-D models cannot handle fractures of arbitrary shape and orientation, however; fully 3-D models are required for this purpose. The literature contains a large number of references to fully 3-D simulations; the majority of these are limited to plane fracture surfaces, however. These are called planar 3-D simulators in this chapter to differentiate them from true fully 3-D simulators which can model out-of-plane fracture growth. Planar 3-D simulators have been developed by Clifton and Abou-Sayed (1979), Barre (1983), Touboul et al. (1986), Morita et al. (1988), Advani et al. (1990), and Gu and Leung (1993). Out of plane 3-D hydraulic fracture growth has been modeled by Lam et al. (1986), Vandamme and Jeffrey (1986), and Sousa et al. (1993). To the authors' knowledge, only Carter et al. (1994), using the predecessor of the simulator described herein, have modeled 3-D fracture in the near-wellbore region of a cased, perforated, and deviated wellbore.

The increasing use of deviated wellbores implies that fully 3-D hydraulic fracture simulators are vital to the petroleum industry. Hydrofracturing is often less effective for deviated wellbores as compared to traditional vertical wells. Some of the problems have been attributed to a poor understanding of the mechanics of fracture initiation and propagation from a deviated wellbore. The complex state of stress which is generated around an inclined wellbore (Yew and Li, 1988; Ong and Roegiers, 1995) means that the fracture propagates with a complex geometry (Behrmann and Elbel, 1990; Hallam and Last, 1991; Weijers and de Pater, 1992; Abass et al., 1996). The complex stress state and fracture geometry can limit the fracture width at the wellbore and hinder the injection of proppant into the fracture leading to premature screenout (Hallam and Last, 1991; Soliman et al., 1996). Nevertheless, the advantages of drilling inclined wellbores are significant. For example, the ability to drill several wells from a single location minimizes production...
Simulating fully 3-D hydraulic fracturing

infrastructure and impact on the environment. Therefore, the ability to model hydraulic fracturing from deviated wells is of ever increasing importance.

In addition to inadequate modeling of the fracture geometry, many of the current hydraulic fracturing simulators do not predict the correct wellbore fluid pressure or fracture geometry even for planar fractures. The proposed reasons for this are numerous (Medlin and Fitch, 1983; Warpinski, 1985; Shlyapobersky et al., 1988; Jeffrey, 1989; Palmer and Veatch, 1990; Johnson and Cleary, 1991; Gardner, 1992; Papathanasiou and Thiercelin, 1993; de Paer et al., 1993 and van den Hoek et al., 1993). The simulator developed here addresses this problem by properly modeling the near crack tip behavior (SCK, 1993). Furthermore, the simulator is ideal for modeling non-planar hydraulic fracturing, and is able to model multiple branching, intersecting, and merging fractures as will be shown in the following section.

21.3 Model Representation

The first key component of an efficient, fully 3-D, hydraulic fracture simulator is the geometric representation of the model. Representation implies computer storage and visualization of the model topology and geometry. This portion of the hydraulic fracture simulator is actually a general purpose, fully 3-D, fracture analysis code, called FRANC3D, under development at Cornell University since 1987 (Martha, 1989; Wawrzynek, 1991; Potyondy, 1993). FRANC3D is capable of modeling multiple, arbitrary, non-planar, 3-D cracks in complex structures, and has pre- and post-processing capabilities for both finite and boundary elements. It relies on a boundary surface representation of the model and a radial edge data structure for storing and accessing topological and geometrical information. It has the ability to do fully automatic or fully user-controlled crack growth simulations, including post-processing of the response information, modifying the geometry, remeshing, and updating the boundary conditions for each stage of crack growth. The complex geometry associated with perforated, cased, and deviated wellbores with multiple non-planar evolving 3-D cracks requires a sophisticated, but easy to use simulation capability. FRANC3D has these capabilities, and some of its individual components are described briefly in the following sections.

21.3.1 Representational model of fracture propagation

Crack growth simulation in FRANC3D is an incremental process, where a sequence of operations is repeated for a progression of models (Figure 21.1). Each step in the

![Figure 21.1 Incremental crack growth simulations; i denotes the increment of crack growth](image)

Figure 21.2 FRANC3D encompasses most of the conceptual crack growth simulation model process relies on previously computed results and represents one crack configuration. There are four primary collections of data, or databases, required for each step. The first is the representational database, denoted \( R_i \) (where the subscript identifies the step). The representational database contains a description of the solid model geometry, including the cracks, the boundary conditions, and the material properties. The representational database is transformed by a discretization (or meshing, \( M \)) process to a stress analysis database \( A_i \). The analysis database contains a complete, but approximate description of the body, suitable for input to a solution procedure (5), usually a finite or boundary element stress analysis program.

The solution procedure is used to transform the analysis database to an equilibrium database \( E_i \) which consists of field variables, such as displacements and stresses, that define the equilibrium solution for the analysis model \( A_i \). The equilibrium model should contain field variables and material state information for all locations in the body, and in the context of a crack growth simulation, should also contain values for stress-intensity factors, or other fracture parameters \( F_i \) for all crack fronts. The equilibrium database is used in conjunction with the current representational database to update \( (U/C) \) the representational model \( R_{i+1} \) including the increment of crack growth as governed by the fracture parameters \( F_i \) and the crack growth function \( C \). This process is performed repeatedly (Figure 21.1) until a suitable termination condition is reached.

FRANC3D encompasses all components of this conceptual model except for the stress analysis procedure (Figure 21.2). The individual components consist of unique databases and functions that operate on the databases, some of which are described in more detail in the following sections.

21.3.2 Solid modeling for crack growth simulations

Simulation of crack growth is more complicated than many other applications of computational mechanics because the geometry and topology of the structure evolve during the simulation. For this reason, a geometric description of the body that is independent of any mesh needs to be maintained and updated as part of the simulation process. The geometry database should contain an explicit description of the solid model including the crack.
The three most widely used solid modeling techniques, boundary representation (B-rep), Constructive Solid Geometry (CSG), and Parametric Analytical Patches (PAP) (Hoffmann, 1989; Mäntylä, 1988; Mortenson, 1985), are capable of representing uncracked geometries. A B-rep modeler stores surfaces and surface geometry explicitly. If implicit topological adjacency information (as defined in the next section) is available as well, two topologically distinct surfaces can share a common geometric description. Cracks, for instance, consist of two surfaces that have the same geometric description; for this reason, among others, a boundary representation was found to be the most suitable of the three modeling techniques for modeling cracks.

21.3.3 Computational topology as a framework for crack growth simulation

Explicit topological information is an essential feature of the representational database for crack growth simulations. The topology of an object is the information about relationships, proximity, and order among features of the geometry – incomplete geometric information. These are the properties of the actual geometry that are invariant with respect to geometric transformations; the geometry can change, but the topology remains the same (Figure 21.3). A topology framework serves as an organizational tool for the data that represents an object and the algorithms that operate on the data.

There are several reasons for using a topological representation for crack growth simulation: (1) topological information, unlike geometrical information, can be stored exactly with no approximations or ambiguity; (2) there are formal and rigorous procedures for storing and manipulating topology data (Mäntylä, 1988; Hoffmann, 1989; Weiler, 1986); (3) any topological configuration can represent an infinite number of geometrical configurations; and (4) topology generally changes much less frequently than the geometry during crack propagation. Investigations into the use of data structures for storing information needed for crack propagation simulations (Wawrzynek and Ingallena, 1987a,b) showed that topological databases were a convenient and powerful organizing agent, and efficient topological adjacency queries make this data structure ideally suited for interactive modeling.

Explicit topological information is used as a framework for the representational database $R_I$ and aids in implementation of the meshing function $M$ and the updating function $U(C)$. In particular, by using a topological database in conjunction with a B-rep modeler, topological entities can serve as the principal elements of the database with geometrical descriptions and all other attributes (such as boundary conditions and material properties) accessed through the topological entities.

Several topological data structures have been proposed for manifold objects: the winged-edge (Baumgart, 1975), the modified winged-edge, the face-edge, the vertex-edge (Weiler, 1985), and the half-edge (Mäntylä, 1988) data structures. However, these data structures cannot be used for modeling fully 3-D hydraulic fracturing because features like bi-material interfaces create non-manifold topologies. Weiler (1986) presented another edge-based data structure for storing non-manifold objects, called the radial-edge, and outlined the corresponding generalized non-manifold Euler operators. The basic topological entities used for modeling are vertices, edges, faces, and regions. An internal crack, for example, consists of vertices, edges, and faces with a null volume region between the crack surfaces. The edge entity is the object through which topological relationships are maintained and queried (Figure 21.4). As the name implies, the edge uses are ordered radially about the edge; each face has two face uses and each face use has a corresponding edge use on the given edge. The radial ordering allows for efficient storage, querying and manipulation of the model topology. As shown in Figure 21.5, this data structure, in addition to bi-material interfaces, is clearly able to represent model topologies consisting of branching or intersecting cracks; both are important features when modeling hydraulic fracturing in a layered rock mass from a cased and deviated wellbore.

A crack is defined within this representational database by both geometry and topology. It consists of multiple surfaces in order to represent the evolving geometry as well as the possibility of intersecting, branching, and merging cracks. Crack surfaces are arranged in pairs (main and mate surface; see Figure 21.5), and each surface is composed of faces, edges and vertices. The edges and vertices are further classified based on their location on the crack surface. For instance, crack front edges represent the leading edge of the crack within the solid. Note that crack growth involves modifying the model topology and geometry to represent newly created fracture surfaces.

![Figure 21.3 Relationship between topology and geometry](image1)

![Figure 21.4 The radial-edge database relies on the radial ordering of edge uses about each edge. A face has two face uses and the edge has a use with respect to each face use (after Weiler, 1986)](image2)
21.3.4 Meshing, crack growth and model update functions

The combination of the boundary representation solid model and the radial edge topological database comprises the representational model $R_i$. Complex 3-D models of deviated, perforated, and cased wellbores including multiple non-planar fractures can be built fairly quickly and easily using this representational model. The other components of the abstract model, such as the discretization (meshing), crack growth and model updating, post-processing, and visualization also take advantage of the topological database. The meshing capabilities (Potyondy et al., 1995) and the crack growth and model update functions (Martha et al., 1993; Carter et al., 1997) in FRANC3D have been described elsewhere. For completeness however, a brief description of both functions is needed here as they relate to modeling hydraulic fracturing.

FRANC3D maintains a consistent geometric representation of the model at each step of propagation. During fracture propagation, the previous crack surface geometry remains the same; new fracture surface is simply added to the model to represent the crack growth (Figure 21.6). There are some exceptions when it is necessary to rebuild the entire fracture geometry, but this is beyond the scope of the present discussion. Therefore, the mesh that is attached to the existing geometric crack surfaces is unaffected by fracture growth because the existing geometry does not change. In truth, the mesh is removed from the geometric crack surface during propagation, but an identical mesh can be regenerated on that surface. A new mesh is attached to the new crack surface. Thus, the process of modeling crack propagation involves neither a 'fixed' nor a 'moving' mesh as described in other hydraulic fracturing literature. The mesh on the existing crack surface can remain fixed or it can be modified, but a new mesh must be added to the new crack surface. Mapping of information from the previous step of propagation to the current step is discussed later.

![Figure 21.5](image) The topology of a typical branching crack

**Figure 21.6** Fracture propagation is modeled by adding new crack surface geometry. The mesh which is attached to the geometry can remain the same for the existing surface

### 21.4 The Physics and Mechanics of Hydraulic Fracturing

The second important component of a robust and accurate, fully 3-D, hydraulic fracture simulator is the ability to properly model the fluid flow coupled with the fracture deformation and propagation. The following discussion is restricted to the framework of linear elasticity and lubrication theory.

#### 21.4.1 Elastic stress analysis

BES is a linear elastic, 3-D, boundary element program (Lutz, 1991). It is based on a direct formulation and uses special hypersingular integration techniques and non-conforming elements on and around the crack surfaces. It is capable of handling multiple loading cases, specifically generating basic solutions (displacements and tractions) for unit tractions at points on the crack surface. Unit tractions are applied to each node on the discretized crack surface. The traction is distributed according to the shape functions of the incident elements, starting at unity at the given node and vanishing to zero at all adjacent nodes.

The displacements at all nodes in the structure are evaluated for each unit traction loading case, providing a matrix of solutions whose generic element $K_{ij}$ is the displacement at node $i$ due to a unit traction at node $j$.

The set of basic solutions is combined to build a single influence matrix which then is used along with the equilibrium fluid pressures to determine the overall structural response due to both the far field boundary conditions and the fluid pressure in the crack. The displacements in the structure can be computed by multiplying each of the basic solutions, obtained for a unit traction at a node, by the fluid pressure at that node. This means that the stress analysis, which is the most time consuming process of the entire simulation, can be performed once for a specific model or crack geometry. Various fluid properties and flow parameters then can be used in the hydraulic fracture simulator based on this single stress analysis.

#### 21.4.2 Fluid flow

A peculiarity of hydraulic fracturing consists of the strong nonlinear coupling between fluid flow and solid deformation, particularly in the vicinity of the fracture front. Proper
Simulating fully 3-D hydraulic fracturing

Coupling, as derived by the Geomechanics Group at Schlumberger Cambridge Research (SCR, 1993, 1994), yields an analytical model for pressure and width near the crack front which corresponds to a stress singularity that is different from the usual Linear Elastic Fracture Mechanics (LEFM) solution. A new term has been coined, Linear Elastic Hydraulic Fracturing (LEHF), to reflect the difference in the solutions. The use of the LEHF analytical model provides an elegant solution to the numerical problem otherwise associated with modeling the front of a hydraulic fracture. (Note that a 3-D crack front becomes a crack tip in 2-D.)

Review of 2-D LEHF solution

Following an approach similar to Spence and Sharp (1985), the Geomechanics Group at SCR (SCR, 1993) has shown that a fluid lag often develops near the crack tip, and this fluid lag negates the influence of the rock fracture toughness. However, by assuming that the fluid reaches the crack tip, a particular singularity develops in both the fluid pressure and the stress field ahead of the crack tip which is unique to hydraulic fracturing (SCR, 1994). This yields an intermediate asymptotic solution for the width and pressure in the crack which is independent of fracture toughness, provided that more energy is dissipated in the fluid than in creating new fracture surface. It is intermediate in the sense that there exists a small region at the very tip of the fracture where a fluid lag develops and LEFM holds, but has little effect on the rest of the solution and is not taken into account.

To obtain the general solution for fluid flow in the vicinity of the tip of a hydraulic fracture propagating in an impermeable solid, the following assumptions are made: crack propagation is self-similar and steady-state, the rock mass is a linear elastic solid in plane strain, lubrication theory is valid, and the fluid is incompressible with a power-law shear-thinning consistency. The boundary conditions include the far-field minimum principal stress, the fracture width at the crack tip, which must be zero, and the fluid velocity at the crack tip where the tip is a moving boundary. Details of how the solution is obtained can be found in SCR (1994). The final expressions for the pressure $p$ and crack opening $w$ in the vicinity of the crack tip, are as follows:

$$ p - p_h = p_h \left[ \frac{2^{\frac{1}{2}}(2 + n')}{\pi(2 - n')} \left( \frac{L_0}{L} \right)^{n'/(2+n')} \left( - \frac{L_0}{\xi} \right)^{n'/(2+n')} \right] $$

$$ w = \frac{1}{2(n'+1)} \int_0^{1} \left( \frac{c_1(n') - c_2(n')}{L} \right) \left( \frac{\xi}{2 + 2n'} \right) $$

$$ L_h = V \left( \frac{K_1'}{E'} \right)^{1/n'} $$

where

$$ p_h = E' \left( \frac{\cos((1 - \alpha)\pi)}{\sin(\alpha\pi)} \right)^{1/n'} \left( \frac{2^{n' + 1}}{n^{2n'(2 + n')}} \right)^{-1/(2 + n')} $$

is a characteristic length particular to hydraulic fracturing, and

$$ \beta = \frac{2}{E} $$

is a characteristic pressure with $\alpha(n') = 2/(2 + n')$. $V$ is the crack tip speed which is equal to the fluid velocity at the tip. $L$ is the fracture half length and $\xi$ is the position from the crack tip. $c_1$ and $c_2$ are constants (SCR, 1994) that are determined numerically; for a Newtonian fluid, they are $c_1 \approx 7.21$ and $c_2 \approx 3.17$. Note that the flow rate at the tip $q_0$ is a function of the speed and the crack opening. $q_0 = V \cdot w = f(V)$.

From these equations, the fluid pressure at the crack tip is found to be singular; for a Newtonian fluid, $n' = 1$ and the order of the singularity is 1/3. The stress at the crack tip has the same order of singularity. The order of the singularity depends on the fluid properties only and, within the assumptions made here, is always weaker than the 1/2 obtained from linear elastic fracture mechanics.

A similar solution was developed for the permeable case and can be found in Lenanach (1995). It involves a supplementary length scale because of the leak-off process and yields a solution which exhibits yet another singularity, also weaker than that of linear elastic fracture mechanics.

Extension of the LEHF solution to 3-D

The behavior of a hydraulic fracture in the vicinity of its propagating front is easily described by the LEHF solution, provided two additional assumptions are made: the crack front is considered to be locally under plane strain conditions, and the local fluid flow parallel to the crack front is negligible. We shall restrict ourselves to the case of a Newtonian fluid, but the process can easily be extended to power-law fluids. The width $w$ in the vicinity of the fracture front is then described by the LEHF solution along any normal to the crack front:

$$ w = (2^{3/6}) \left( \frac{\mu V}{E} \right)^{1/3} \rho^{1/3} = BV^{1/3} \quad (21.5) $$

where $\beta = (2^{3/6}) \left( \frac{\mu}{E} \right)^{1/3} \rho^{1/3}$, and $\rho$ is the curvilinear distance, measured on the fracture surface, between any point and the fracture front. $\mu$ is the fluid viscosity.

The equation describing the mass conservation within the bulk of the fracture can then be transformed in order to accommodate the introduction of the LEHF solution. The fracture is considered as a surface in 3-D space $F$. Consider a subdomain $\Omega$ of the fracture, characterized by a boundary $\Gamma$, a pressure field $p$, an aperture field $w$ and fluid flux field $q$. $t$ is the time. If one considers any closed domain $A$ included in $\Omega$ (Figure 21.7), fluid enters and leaves $A$ through its boundary $\partial A$. Fluid is stocked by an increase of the fracture width $w$ with time $t$.

Considering also a source term at a point $O$ included in $A$ (injection or sink of value $Q(t)$), one then can write for an incompressible fluid:

$$ \int_A q \cdot n dA + \int_A \frac{\partial w}{\partial t} dA = Q(t)w_O \quad (21.6) $$
After transformation of the contour integral into a surface integral, one gets
\[ \int_A \left( \frac{\partial w}{\partial t} + \text{div} q - Q(t) \delta(\Omega) \right) dA = 0 \] (21.7)
where \( \delta \) is the Dirac operator. This equation must hold whatever the chosen domain \( A \), which yields the mass conservation equation for any point inside the considered domain \( \Omega \):
\[ \frac{\partial w}{\partial t} + \text{div} q = Q(t) \delta(\Omega) \] (21.8)

One can write a variational form for Equation (21.8) by considering an auxiliary pressure field \( \delta p \):
\[ \int_\Omega \delta p \frac{\partial w}{\partial t} d\Omega + \int_\Omega \delta p \text{div} q d\Omega = Q(t) \delta p(\Omega) \] (21.9)

Using \( s \) as the curvilinear abscissa along the boundary \( \Gamma \) and \( n_s \) as the outward normal to \( \Gamma \) at point \( s \) (Figure 21.7), one can apply the divergence theorem to the second integral of Eq. (21.9), to get
\[ \int_\Omega \delta p \text{div} q d\Omega = - \int_\Omega q \cdot \text{grad}(\delta p) d\Omega + \int_\Gamma \delta p (q \cdot n_s) ds \] (21.10)

The lubrication approximation for a Newtonian fluid of viscosity \( \mu \) gives
\[ q = -\frac{w^3}{12\mu} \text{grad} p \] (21.11)

Setting \( q \cdot n_s = q_b \) (Figure 21.7) and plugging the lubrication equation into Eq. (21.9), it becomes
\[ \int_\Omega \delta p \frac{\partial w}{\partial t} d\Omega + \int_\Omega \left( \frac{w^3}{12\mu} \text{grad} p \right) \cdot \text{grad} \delta p d\Omega + \int_\Gamma \delta p q_b d\Gamma = Q(t) \delta p(\Omega) \] (21.12)

As illustrated in Figure 21.7, it is convenient to choose \( \Omega \) so as to describe the bulk of the fracture, whereas \( F - \Omega \) corresponds to the domain of the fracture in the vicinity of the

**Finite element implementation of fluid flow coupled with the structural response**

The finite element method is used to solve the fluid flow equation coupled with the structural response. The special form of the fluid flow equation developed in Section 21.4 allows one to separate the fracture surface into two regions: the bulk of the fracture \( \Omega \) where width and pressure are to be determined and regular shell finite elements are used, and the near vicinity of the crack front where special tip elements adjacent to the crack front are used. Thanks to the form of Eq. (21.14), the special tip elements only represent the LEHF solution, and nodal width and pressure are never computed, except at the boundary \( \Gamma \) between the regular elements and the tip elements.

### 21.5.1 Finite element formulation of fluid flow equation

Considering a set of \( n \) shape functions \( N \) over the domain \( \Omega \), the pressure can be approximated in the finite element sense by \( p = \sum_{i=1}^{in} N_i \hat{p}_i \), where \( \hat{p}_i \) is the pressure at node \( i \) (or in matrix form \( p = N^T \hat{p} \)). The small increment in pressure \( \delta p \) is approximated in a similar way \( \delta p = \sum_{i=1}^{in} N_i \delta \hat{p}_i \). These two expressions, when introduced into (Eq. 21.14), lead to
\[ \int_\Omega \left( \sum_{i=1}^{in} N_i \delta \hat{p}_i \right) \frac{\partial w}{\partial t} d\Omega + \int_\Omega \frac{w^3}{12\mu} \text{grad} \left( \sum_{j=1}^{jm} N_j \hat{p}_j \right) \cdot \text{grad} \left( \sum_{i=1}^{in} N_i \delta \hat{p}_i \right) d\Omega \]
\[ + \int_\Gamma \left( \sum_{i=1}^{in} N_i \delta \hat{p}_i \right) \beta V^4/3 d\Gamma = Q(t) \left( \sum_{i=1}^{in} N_i(O) Q(t) \right) \] (21.15)

Requiring stationarity of Eq. (21.15) leads to a set of \( n \) differential equations. Introducing the finite element approximation for the width \( w \) together with the time discretization, the
\[ \sum_{j=1}^{m} N_j \left( \sum_{k=1}^{k_{\text{max}}} N_k \hat{w}_k \right) \mathrm{grad}N_i \cdot \mathrm{grad}\left( \sum_{j=1}^{m} N_j \hat{p}_j \right) \, d\Omega \]

where \( \hat{w}_j \) is the value of the width \( w \) at node \( j \).

For simplicity, the \( i \)th shape function is associated with the node at which it takes a value of 1. It is worth noting that a shape function \( N_i \) is equal to zero when the point considered for the integration is not part of an element containing the associated \( i \)th node. Therefore, the integrals in Eq. (21.16) can be reduced to integrals over a single element \( \Gamma_i \):

\[ \int_{\Omega_i} N_i \left( \sum_{j=1}^{m} N_j \left( \frac{\hat{w}_j (t_{n+1}) - \hat{w}_j (t_n)}{t_{n+1} - t_n} \right) - \hat{w}_j \right) \, d\Omega \]

where \( \Gamma_i \) is the portion of the boundary \( \Gamma \) which intersects the element \( \Omega_i \). Two cases can be considered, depending upon the location of the element \( \Omega_i \) with respect to the boundary \( \Gamma \).

### 21.5.3 Element adjacent to \( \Gamma \)

If \( \Gamma_i \) is not void (the element is adjacent to the boundary), the second and third terms of Eq. (21.17) are to be modified. The pressure \( \hat{p}_i \) at each node belonging to the boundary \( \Gamma_i \) satisfies

\[ \hat{p}_i = \hat{P}_i - E^{2/3} \left( \frac{\mu}{3 \hat{P}_i} \right)^{1/3} \hat{V}_i^{1/3} \]  

Note that this equation simply means that the derivative of the pressure normal to the boundary is known. Assuming that there are \( n \) nodes in the considered element, \( m \) of which are not on the boundary, the second term \( I_i \) of Eq. (21.17) can be transformed into:

\[ I_i = \int_{\Gamma_i} \frac{1}{12 \mu} \left( \sum_{k} N_k \hat{w}_k \right)^3 \mathrm{grad}N_i \cdot \mathrm{grad}\left( \sum_{j=1}^{n} N_j \hat{p}_j \right) \, d\Omega \]

where \( \hat{B}_j = E^{2/3} \left( \frac{\mu}{3 \hat{P}_j} \right)^{1/3} \).

Finally,

\[ I_i = \int_{\Gamma_i} \frac{1}{12 \mu} \left( \sum_{k} N_k \hat{w}_k \right)^3 \mathrm{grad}N_i \cdot \mathrm{grad}\left( \sum_{j=1}^{n} N_j \hat{p}_j + \sum_{j=m+1}^{n} N_j (\hat{P}_j - \hat{B}_j \hat{V}_j^{1/3}) \right) \, d\Omega \]  

(21.20)

For convenience, \( \hat{P}_j \) can be lumped with \( \hat{p}_j \) (note that \( \hat{p}_j \) is not the true pressure at the nodes sitting on \( \Gamma_i \)), leading to a more compact notation

\[ I_i = \int_{\Gamma_i} \frac{1}{12 \mu} \left( \sum_{k} N_k \hat{w}_k \right)^3 \mathrm{grad}N_i \cdot \mathrm{grad}\left( \sum_{j=1}^{n} N_j \hat{p}_j \right) \, d\Omega \]

(21.22)

Assuming \( l \) nodes along \( \Gamma_i \), one can also define a finite element approximation for the flow \( q_b \) of the fluid normal to \( \Gamma_i \). For the case of a Newtonian fluid:

\[ q_b = \sum_{k=1}^{l} M_k \hat{B}_k \hat{V}_k^{1/3} \]

(21.23)
where \( \dot{V}_k \) is the speed at node \( k \), \( \hat{\rho}_k \) is the evaluation of \( \dot{\beta} \) at node \( k \), and \( M \) is a set of shape functions. Then, the third term of Eq. (21.17) becomes:

\[
\int_{\Omega} N_i \left( \sum_{k=1}^{k_{max}} M_k \hat{\rho}_k \dot{V}_k^{4/3} \right) d\Omega = \int_{\Gamma} N_i \left( \sum_{k=1}^{k_{max}} M_k \hat{\rho}_k \dot{V}_k^{4/3} \right) d\Gamma
\]  
(21.24)

where \( \hat{\rho}_k \) is the curvilinear distance from the fracture tip to node \( k \).

If the various nodal speeds are known, the \( i \)th equation can now be reordered as follows:

\[
\int_{\Omega} N_i \left( \sum_{j=1}^{j_{max}} N_j \left( \hat{w}_j(t_{n+1}) - \hat{w}_j(t_n) \right) / t_{n+1} - t_n \right) d\Omega + \sum_{j=1}^{j_{max}} \hat{p}_j \int_{\Omega} \frac{1}{12\mu} \left( \sum_{k=1}^{k_{max}} N_k w_k \right)^3 \nabla N_i \cdot \nabla N_j d\Omega
\]

\[
= \int_{\Omega} N_i \left( \sum_{k=1}^{k_{max}} M_k \hat{\rho}_k \dot{V}_k^{4/3} \right) d\Omega
\]

\[
= Q(t)N_i(O)
\]  
(21.25)

Equations (21.18) and (21.25) were implemented in FRANC3D in order to model fluid flow in the fractures while accounting for the near crack tip behavior.

21.5.4 Coupling with the structural response of the rock

To adequately consider the coupling between elasticity and fluid flow, both the structural and the fluid flow equations are solved at the same time for both the nodal widths and fluid pressures.

Linear elasticity provides the following linear relation between the fracture apertures \( w \) and the fluid pressure \( p \)

\[
w = w_p + w_o
\]

\[
w = G p + w_o
\]  
(21.26)

where \( w_o \) is the contribution from the external stresses and \( G \) is the influence function relating the fracture opening due to the internal fluid pressure \( (w_p) \) to the fluid pressure in the fracture. This relation can be expressed as a set of equations using the flexibility matrix \( K \) computed by BES (see previous section). For the nodes which are located neither on the crack front nor on \( \Gamma \), the \( i \)th equation of this set can be written as

\[
\sum_{j} K_{i,j} \hat{p}_j - \dot{w}_i = w_o
\]  
(21.27)

where \( w_o \) is the nodal width induced by the external stresses. For nodes located on the crack front, the width is set to zero. The fluid pressure is set to zero also in order to conform with the assumed existence of a fluid lag; therefore this relation disappears at the crack front. For the pressure at the nodes located on \( \Gamma \), there is an extra term to be taken into account and the equation becomes:

\[
\sum_{j} K_{i,j} \hat{p}_j - \dot{w}_i = \sum_{k} K_{i,k} \hat{\beta}_k \dot{V}_k^{4/3} - w_o
\]  
(21.28)

When solving for both nodal width and pressure, a strong non-linearity arises in the set of fluid flow equations due to the cubic power of the crack opening displacement \( w \). This is handled by considering that, for every term in which \( w \) arises, the value of \( w \) will be fixed and equal to either the value of the width at the previous iteration or, for the first iteration of any time stage, the value of the width at the previous time stage.

One can then reorder the \( i \)th equation of the set of fluid flow equations as

\[
\sum_{j=1}^{j_{max}} \frac{\hat{w}_j}{\Delta t} \int_{\Omega} N_i N_j d\Omega + \sum_{j=1}^{j_{max}} \hat{p}_j \int_{\Omega} \frac{1}{12\mu} \left( \sum_{k=1}^{k_{max}} N_k w_k \right)^3 \nabla N_i \cdot \nabla N_j d\Omega
\]

\[
= \sum_{j=1}^{j_{max}} \frac{\hat{w}_j(t_n) - \hat{w}_j(t_{n+1})}{\Delta t} \int_{\Omega} N_i N_j d\Omega + \int_{\Gamma} N_i \left( \sum_{k=1}^{k_{max}} M_k \hat{\rho}_k \dot{V}_k^{4/3} \right) d\Gamma + Q(t)N_i(O)
\]  
(21.29)

This results in a set of \( n \) fluid flow equations and \( n \) structural equations with \( n \) unknown nodal widths and \( n \) unknown nodal fluid pressures. The solution process is described in more detail in Section 21.7.

21.6 Extensions to the Model and Formulation

Current trends in research and in field applications are pushing development of the simulator towards being able to model indirect vertical fracturing (with fracture containment, pinching, closure and fluid recession as corollaries), sand control (with screen-out as a corollary) and steam injection. It could also be used as a tool to study the need for special completions (with the ability to handle multiple fractures from cased and inclined wellbores as a corollary). Some of the features that are needed to model these problems include: (1) partial propagation of a fracture front across material interfaces; (2) simultaneous solution of crack surface contact tractions and fluid flow pressures during fracturing of layered formations with differing stresses and stiffnesses; (3) branching, intersecting, and merging cracks (for example, consider T-shaped fractures that can form at material interfaces); (4) proppant transport; and (5) bi-material interface slip and pressurization. As a step towards these developments, BES has been modified recently to run on parallel computers such as the IBM SP2 and the SG Power Challenge (Shah et al.,
1997), allowing larger more complex problems to be modeled with increased accuracy and efficiency.

Unlike the subjects mentioned above, each a research topic in itself, a fairly simple extension to the simulator can be made in order to model hydraulic fracture propagation in the presence of a porous and permeable rock mass where fluid is able to leak out of the fracture into the surrounding rock.

21.6.1 Introduction of leak-off

Carter’s 1-D formulation (Carter, 1957), which uses a single lumped leak-off coefficient $C_l$, is the standard model in oil field applications for taking into account fluid leaking away from the fracture into the formation. It is a simple model of fluid diffusion through the fracture walls. The underlying assumptions are that the outward flow is normal to the direction of fracture propagation and that variations in the difference between the fluid pressure in the fracture and the pore pressure remain small.

The Carter leak-off model takes the form:

$$u_l = \frac{2C_l}{\sqrt{\tau(x)}}$$  (21.30)

where $u_l$ is the fluid velocity of the leaking fluid, and $\tau$ is the exposure time at point $x$. Note that the factor 2 in front of $C_l$ corresponds to fluid leaking through the two faces of the fracture, where $C_l$ is an empirical constant related to the porosity and permeability of the rock.

This introduces an extra term in the mass balance equation, which now becomes

$$\frac{\partial w}{\partial t} + \text{div} q + \frac{2C_l}{\sqrt{\tau(x)}} = Q(t) \delta(\Omega)$$  (21.31)

This impacts the finite element formulation of the fluid flow described in Section 21.5. An extra term has to be added on the right-hand side of Eq. (21.29). For the $i$th equation, this supplementary term is:

$$- \int_{\Omega_i} N_i \left( \frac{2 \sum_j N_j C_{lj}}{\sqrt{\tau_n + \Delta \tau - \sum_j N_j \theta_j}} \right) d\Omega$$  (21.32)

where $C_{lj}$ is the value of the leak-off coefficient at node $j$ and $\theta_j$ is the time at which the fluid reached node $j$.

The presence of leak-off also modifies the behavior in the vicinity of the tip. The pressure and width tip fields are now governed by a new series of equations (see Lenoach, 1995):

$$w = 6.6687 \left( \frac{\mu C_l}{E'} \right)^{1/6} V^{1/6} \rho^{5/6} + 2.2545 \left( \frac{\mu}{C_l E'} \right)^{1/2} V^{1/2} \rho^{3/4}$$  (21.33)

$$p(\rho) = p_0 - 0.43E^{3/4}(\mu C_l)^{1/4} V^{1/2} \rho^{-3/8} - 0.42E^{1/2}(\mu C_l)^{1/2} V^{3/4} \rho^{-1/4}$$  (21.34)

with similar notations as in Section 21.4. $p_0$ is a constant dependent upon fracture geometry.

This changes the expression for the flow $q = V \cdot w = f(V)$ across the boundary $\Gamma$ between regular elements and tip elements. Introducing the variational formulation for the fluid flow equation, the right-hand side term corresponding to the flow across the boundary still is

$$\int_{\Gamma} \delta p(q^i \cdot n) ds = \int_{\Gamma} \delta p q_b ds$$  (21.35)

Introducing the new expression for the width as given by Eq. (21.33) and the finite element discretization, this becomes for the $i$th equation:

$$\int_{\Gamma_i} N_i(\alpha \gamma^{9/8} + \gamma \nu^{7/2}) d\Gamma = \int_{\Gamma_i} N_i \left[ \sum_j N_j \alpha_j \left( \sum_k N_k \tilde{\nu}_k \right)^{9/8} + \left( \sum_j N_j \gamma_j \right) \left( \sum_k N_k \tilde{\nu}_k \right)^{7/4} \right] d\Gamma$$  (21.36)

$$\alpha_i = 6.6687 \left( \frac{\mu C_l}{E'} \right)^{1/6} V^{1/6} \rho^{5/6}$$  (21.37)

$$\gamma_i = 2.2545 \left( \frac{\mu}{C_l E'} \right)^{1/2} V^{1/2} \rho^{3/4}$$  (21.38)

where $E'$ and $C_l$ are also evaluated at point $i$.

It is clear that $\alpha$ tends to zero, whereas $\gamma$ tends to infinity when the leak-off coefficient tends to zero. In short, the permeable solution does not tend in a clear manner towards the impermeable solution when the leak-off coefficient tends to zero. It is, however, clear physically and verified numerically that the width field corresponding to a very small amount of leak-off is extremely similar to that described by the impermeable solution (Eq. (21.5)).

A criterion, therefore, is needed to switch from the permeable to the impermeable solution when the amount of leak-off is small. The permeable solution actually relies on a parameter $\eta$ to be small (Lenoach, 1995), where $\eta$ is given by

$$\eta = \left( \frac{\mu}{E'} \right)^{1/3} \left( \frac{V}{4C_l} \right)^{2/3}$$  (21.39)

As the exponent in $\rho$ is different for the two solutions (impermeable and permeable), no absolute criterion can easily be established. It was determined empirically that as soon as $\eta$ was greater than 1/20, one could safely switch from the permeable solution to the impermeable solution. In order to be able to mix the two solutions, two nodal switch functions $G(x_i)$ and $H(x_i)$ were introduced: so that $G = 0$ and $H = 1$ if $\eta(x_i) < 1/20$ (permeable case), and $G = 1$ and $H = 0$ otherwise (impermeable case).

A mixed sink term is then

$$\int_{\Gamma_i} N_i \left[ \sum_j H_j N_j \alpha_j \left( \sum_k H_k N_k \tilde{\nu}_k \right)^{9/8} + \left( \sum_j H_j N_j \gamma_j \right) \left( \sum_k H_k N_k \tilde{\nu}_k \right)^{7/4} \right] d\Gamma$$  (21.39)
Simulating fully 3-D hydraulic fracturing

\[ \alpha_t = 6.6687 \left( \frac{\mu C_t}{E} \right)^{1/4} \left( \frac{\rho}{\rho_0} \right)^{5/8} \]

\[ \beta_t = 7.206 \left( \frac{\mu}{E} \right)^{1/3} \left( \frac{\rho_0}{\rho} \right)^{2/3} \]

\[ \gamma_t = 2.255 \left( \frac{\mu}{C_t} \right)^{1/2} \left( \frac{\rho}{\rho_0} \right)^{3/4} \]

(21.40)

This approach was implemented and was found to function well. The only drawback it presents is the necessity to gradually increase the leak-off coefficient from zero to its desired value during the solution process for the first time stage in order to preserve reasonably quick convergence of the algorithm.

21.7 Solution Procedure

Hydraulic fracture propagation is a nonlinear, time-dependent, moving boundary problem that involves simultaneous satisfaction of solid deformation, fluid flow and fracture mechanics. Following the discretization in both time and space, the solution consists of a series of ‘snapshots’ that correspond to unique instances in time and crack shape. Two approaches can be followed to obtain the next term in such a series: one can either fix the time step and look for the corresponding geometry, or fix the geometry and look for the corresponding time. Although the first approach is more intuitive, the latter scheme was chosen as it minimizes the amount of computation. Note that the crack initiation process is not modeled; rather a model with an initial starting crack is the beginning point for the simulation. The mechanics of fracture initiation and breakdown pressure are beyond the scope of this chapter.

21.7.1 Computing the solution for a particular model

The special case of the initial solution for the starting crack will be detailed in section 21.7.2. For subsequent time stages (actually fracture geometry stages), it is assumed that the solution at the previous time stage \( t_n \) is known. It consists of crack geometry \( \Omega(t_n) \), crack volume \( V_0(t_n) \), a set of nodal widths \( \bar{w}_j(t_n) \), a set of nodal pressures \( \bar{p}_j(t_n) \), and a set of nodal crack front speeds \( \bar{V}_j(t_n) \). Assuming the geometry of the current model was computed from the previous solution, the process which leads to the solution of the current model is complicated and has been separated into three subsections for clarity.

Meshing and updating information

As described in section 21.3, crack propagation involves a change in the model geometry. The mesh that is attached to the crack surface is merely a mathematical representation of the true geometry. For a given stage of fracture propagation, the previous, \( \Omega(t_n) \), and current, \( \Omega(t_{n+1}) \), fracture geometries are known. The current fracture surface contains the previous surface plus some new surface representing the propagation. A mesh is attached to the geometry surface; in general, the mesh on the portion of the current fracture surface that was the previous fracture surface can remain fixed or can be modified (to reduce the number of elements for example). The set of nodal widths and pressures from the previous solution must be mapped onto the mesh of the current fracture. This is a trivial procedure if the mesh has remained fixed. If the mesh has been modified, the nodal widths and pressures for the new mesh must be interpolated from the previous mesh. Efficient algorithms to find the location of the node on the previous mesh and to interpolate the response have been written for this purpose (see Pomyody, 1993), but will not be discussed here.

Computation of the flexibility matrix

From the fracture geometry \( \Omega(t_{n+1}) \), the elastic properties of the model and the corresponding boundary conditions, BES is used to compute the flexibility matrix as described in section 21.4.2. Using the flexibility matrix, one can convert the nodal pressures inside the fracture into nodal fracture widths. This reflects both the influence of the geometry of the fracture, the geometry of the model, the elastic properties of the model, and the boundary conditions on the model. The computation of the elastic flexibility matrix is by far the most time consuming part of the analysis, and guided the earlier choice of fixing the geometry instead of the time.

Iterative solution of the fluid flow

The results from the previous time stage, \( t_n \), are used as starting values for the solution process. Given an initial set of values for crack opening displacement, fluid pressure and crack front speed, the iterative scheme proceeds at two levels. First, the set of nonlinear equations describing the fluid flow inside the fracture coupled to the structural response of the model is solved in an iterative manner. Once convergence is achieved based on the crack opening displacements, the global mass balance and the crack front speeds are considered in the following way to determine the absolute time corresponding to the current geometry. The total volume of fluid injected should be equal to the volume of the fracture minus the volume of fluid that has leaked away into the formation. Also, the fluid speed at each point of the crack front should equal the crack propagation speed, i.e., the crack advance divided by the time step. Note that, within the framework used here, these two relationships both express satisfaction of global mass balance. If these two relationships are not satisfied, the time step is adjusted and the fluid flow equation is solved iteratively again. This process continues until the solution has converged on both the nodal values and the time, thus satisfying both elasticity and fluid flow.

Fracture propagation

Before a crack can be propagated, the total model displacements must be determined. This is done by combining the displacements due to the far-field applied loading with the displacements produced by the fluid pressurization in the fracture.

The process of propagating a crack in FRANC3D has been described elsewhere (Martha et al., 1993; Carter et al., 1997). The only difference in this case is that the crack is
This provides the initial values to be used for the real fracture geometry, and the solution scheme then proceeds as described in Section 21.7.1.

Two solutions have been developed covering most initial crack geometries in hydraulic fracturing, one for a slot crack and one for a penny-shaped crack (or radial crack). Following the approach of the SCR Geomechanics Group (SCR., 1994), the main assumption behind these solutions is that the pressure profile near the tip is governed by a singular field, and that the resulting stress intensity factor is zero. This allows one to obtain a pressure profile from which all other quantities are derived. These solutions, therefore, produce time, crack opening displacements, crack front speeds and fluid pressure inside the crack as a function of fracture extension, elastic parameters, and pumping parameters.

21.8 Verification and Validation

One of the most important aspects in the development of any numerical simulator is the verification of the approach as well as the computer software. To verify this 3-D simulator, one must analyse problems for which the solution is known. In this case, the fully 3-D simulator is used to model simple 2-D fractures: a KGD and a radial hydraulic fracture. The results from the 3-D simulator are compared with those of a previously verified 2-D simulator. Furthermore, the 3-D simulator is used to model some experiments that were performed at Delft University which provides additional validation and shows the capability of the simulator to model fully 3-D hydraulic fractures in the near-wellbore region.

21.8.1 Modeling hydraulic fractures of simple geometries

FRANC3D has been compared with a hydraulic fracturing model (Desroches and Thiercelin, 1993) which can only model simple geometries such as KGD and radial, but considers full coupling of fluid flow, elastic deformation and fracture propagation. It includes a complete description of the fracture tip, in particular the effect of fracture toughness and the possibility for the fracturing fluid not to reach the tip of the fracture ("fluid lag"). This model, referred to as Loramec, has been validated using laboratory experiments (de Pater et al., 1996). Although the formulation of these two models is radically different, the underlying physical assumptions are similar so they should yield similar results for a similar fracture geometry.

To simulate a plane strain geometry with FRANC3D, a small slot crack was introduced in a cube (Figure 21.9). The fracture was propagated with an evenly distributed line source. Data for comparison with Loramec were taken in the middle of the crack. The relevant parameters were: Young’s modulus of 24.1 GPa, Poisson’s ratio of 0.02, fracture toughness of 0.25 MPa√m, fluid viscosity of 130,000 cp, flow rate/height of 5.92 x 10^{-9} m^3/s and closure stress of 9.7 MPa.

Figure 21.10(a) shows the fracture extension versus time for Loramec and FRANC3D together with a profile of pressure (Figure 21.10(b)) and width (Figure 21.10(c)) from the fracture mouth to the fracture tip. Excellent agreement between FRANC3D and Loramec is observed. Note that the FRANC3D model used only six linear elements along the fracture extension in addition to the last tip element. This demonstrates how well the tip element captures the essence of the solution, allowing one to use a coarse mesh away from the tip.
To simulate an axisymmetric (radial) geometry, a penny-shaped crack was introduced in the center of a cube (Figure 21.11). The size of the crack was two orders of magnitude smaller than that of the cube to simulate an infinite medium. The fracture was propagated using a point source at the center of the crack. Data for comparison with Loramec were taken along a crack radius. The relevant parameters were similar to those for the KGD geometry except for the injection rate of $2.96 \times 10^{-9} \text{m}^3/\text{s}$. These parameters correspond to those of one of the laboratory experiments that was used to validate the Loramec model. Figure 21.12(a) shows the fracture radius versus time for Loramec and FRANC3D, together with a profile of pressure (Figure 21.12(b)) and width (Figure 21.12(c)) from the fracture center to the fracture tip. There is excellent agreement between the two solutions for this model as well.

21.8.2 Modeling experimental data

To study the 3-D geometry of hydraulic fractures, a matrix of hydraulic fracturing experiments using a true triaxial loading frame was carried out on physical models at
the University of Delft (Weijs, 1995). The simplest configuration, an open-hole model was chosen for comparison with FRANC3D here so that the extra complexity due to the perforations and casing could be avoided. Simulations of the other test configurations are on-going (Shah et al., 1997), but cannot be described here due to time and space limitations.

The geometry of the chosen configuration, COH10, is presented in Figure 21.13. A cylindrical wellbore of radius 1 cm was drilled in a 30 cm edge cube. The wellbore was drilled parallel to the direction of the minimum stress, perpendicular to the preferred fracturing plane. The experimental parameters were: Young’s modulus E = 35 GPa, Poisson’s ratio ν = 0.15, fracture toughness $K_{t_c} = 0.6$ MPa√m, fluid viscosity $\mu = 100000$ cp, injection rate $Q = 0.088$ cc/min, $\sigma_{1,2,3} = 23, 12.1$ and $9.7$ MPa, respectively. Splitting of the blocks after the experiment showed that a single fracture initiated parallel to the wellbore and slowly turned to propagate perpendicularly to the wellbore. The pressure in the wellbore was recorded during the experiment. Two displacement transducers DIS2 and DIS3 were set 90° apart in the middle of the wellbore to measure the deformation of the wellbore. DIS2 measures the deformation in the direction of the maximum principal
stress. The output of FRANC3D was compared against both this data and the radius of reorientation of the fractures, which was measured after dismantling the blocks.

Although categorized as 'open-hole', there was still a borehole assembly in this experiment, which consisted of two 10 cm long packers glued at the two extremities of the wellbore, leaving an open-hole section of 10 cm at the center of the wellbore. To simulate the wellbore assembly that was used during the experiment, the cylinder representing the wellbore was split into three regions; the middle section corresponds to the open-hole
section whereas the other two correspond to the packer arrangement. As the packer assembly was glued to the wellbore, it was considered as a rigid inclusion and was modeled by a zero normal displacement boundary condition on the packer zone. In the open-hole section, two initial slot fractures were added, 180° apart. Their location was taken as indicated by the experiments; the fracture initiation process was not modeled.

Figure 21.14 shows the comparison between the data obtained from the experiment and the output of FRANC3D. The comparison is made in terms of wellbore pressure (Figure 21.14(a)) and displacements of the wellbore wall as measured by the two LVDT sensors DIS2 (Figure 21.14(b)) and DIS3 (Figure 21.14(c)). Apart from numerical noise, the pressure predicted by FRANC3D is comparable to that measured in the laboratory and the two horizontal displacements are well predicted. As DIS3 is an indirect measurement of the fracture aperture, this indicates that the fracture aperture at the wellbore is well predicted by the model. The only discrepancy between the experiment and the output of the model is the radius of reorientation of the fracture; the experiment showed that the fracture was slow to turn (about 10 cm for the radius of reorientation (Weijers, 1995)), whereas the simulated fracture turned much more quickly towards the preferred fracturing plane (a radius of reorientation of about 2 cm was computed). It is proposed that this is due to some influence of the packer assembly that is not taken into account by our modeling (de Pater, personal communication, 1996).

21.9 Conclusions

It is necessary to have a fully 3-D simulator to correctly model some of the problems regularly encountered in the field during hydraulic fracturing, such as the near wellbore fractures of a deviated well. If a robust and accurate simulator is available, improvements in oil/gas recovery are possible as well as savings in time and money during completion and re-injection.

A simulator that is capable of modeling multiple, fully 3-D, non-planar hydraulic fractures has been built. It maintains a consistent geometric representation of the fracture geometry throughout the analysis and properly couples the fluid flow in the fracture with the fracture deformation. It is verified using previous solutions for simple 2-D crack geometries as well as 3-D experimental results. Enhancements and additions are in progress to allow more complex problems to be modeled and to increase the speed, efficiency, and accuracy of the simulator.

An additional advantage of the software is that it is easily adapted to other problems of fracture in geomechanics, such as pressure grouting of cracks in dams (Ingraffea et al., 1995), cohesive crack propagation (Carter et al., 1993), and compression induced fracture from inclined flaws in brittle materials (Germanovich et al., 1996).

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Simulating fully 3-D hydraulic fracturing


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